Tangent Cones

4

In this chapter certain approximations of sets are considered which are very useful for the formulation of optimality conditions. We investigate so-called tangent cones which approximate a given set in a local sense. First, we discuss several basic properties of tangent cones, and then we present optimality conditions with the aid of these cones. Finally, we formulate a Lyusternik theorem.

4.1 Definition and Properties

In this section we turn our attention to the sequential Bouligand tangent cone which is also called the contingent cone. For this tangent cone we prove several basic properties.

First, we introduce the concept of a cone.

Definition 4.1 (cone).

Let C be a nonempty subset of a real linear space X.

(a) The set C is called a *cone* if

$$x \in C,\ \lambda \geq 0 \implies \lambda x \in C$$

(compare Fig. 4.1).

J. Jahn, *Introduction to the Theory of Nonlinear Optimization*,
https://doi.org/10.1007/978-3-030-42760-3_4

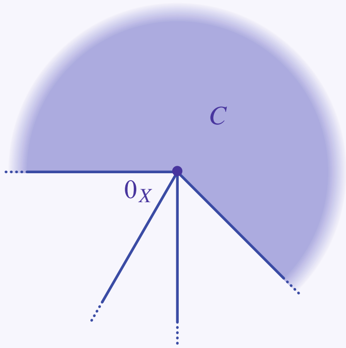

Fig. 4.1 Cone

(b) A cone C is called *pointed* if

$$x \in C, \; -x \in C \implies x = 0_X$$

(compare Fig. 4.2).

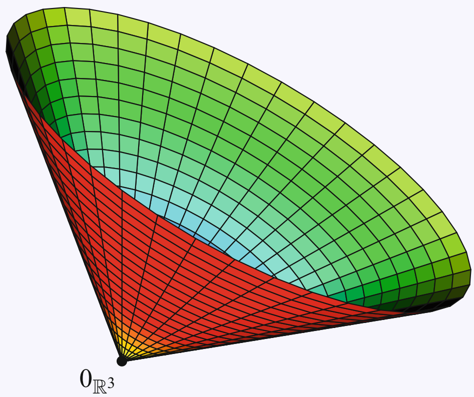

Fig. 4.2 Pointed cone

Example 4.2 (pointed cone).

(a) The set

$$\mathbb{R}^n_+ := \{x \in \mathbb{R}^n \mid x_i \geq 0 \text{ for all } i \in \{1, \ldots, n\}\}$$

is a pointed cone.